

# Search for Non-Standard Model CP/T Violation at Tau-Charm Factory

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## Abstract

We systematically investigate the possibility of finding CP/T violation in the  $\tau$  sector at Tau-Charm Factory. CP/T violation may occur at  $\tau$  pair production process, expressed as electric dipole moment, and at *tau* decay processes. By assuming that electric dipole moment as large as  $10^{-19}$ e-cm and CP/T violation effect originating from  $\tau$  decay as large as  $10^{-3}$  are observable at Tau-Charm Factory, we studied all the possible extensions of the SM which are relevant for generating CP/T violation in  $\tau$  sector. And we pointed there are a few kind of models, which are hopeful for generating such CP/T violation. For these models we consider all the theoretical and current experimental constraints and find that there exists some parameter space which will result in a measurable CP/T violation. Therefore we conclude that Tau-Charm Factory is a hopeful place to discover CP/T violation in  $\tau$  sector.

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# 1 Introduction

The origin of CP violation has remained an unsolved problem since the discovery of CP violation in K meson system a quarter ago[1]. Although the observed CP violation in K meson system can be accommodated in the standard model (SM) of electroweak interactions by virtue of a physical complex phase in the three by three Cabibbo-Kobayashi-Maskawa matrix (CKM) [2], it is not clear if CKM mechanism is really correct or the only source for CP/T violation [3]. To verify CKM mechanism one needs not only the information on K meson mixing and decay but also that from the B meson system or other systems. The main physical purpose of B factory is to test the CKM mechanism. However even if CKM is the correct mechanism to describe the CP violation in K and B meson mixing and decay, it is not necessary that the CKM matrix is the only source of CP/T violation in the nature [4]. As pointed out by Weinberg [5], unless the Higgs sector is extremely simple, it would be unnatural for Higgs-boson exchange not to contribute to CP/T non-conservation. CKM matrix may explain the observed CP violation in K meson system and possibly the CP violation in B meson system, while other new sources of CP/T violation may occur everywhere it can. In fact there are some physical motivations for people to seek the new sources of CP/T violation. One motivation is from strong CP problem in the SM [6]. For most of the scenarios to solve this problem they need more complex vacuum structure and therefore new CP non-conservation origin. Another motivation is from cosmology, most astrophysical investigation shows that the additional sources of CP violation are needed to account for the baryon asymmetry of universe at present [7]. The third motivation is from supersymmetry. Even in the minimal supersymmetrical standard model (MSSM), there are some additional CP non-conservation sources beyond the CKM matrix [8]. Now the question is at what places the possible new CP/T violation effects may show up and what is the potential to search for those effects. In this work we are going to study systematically

on the possibility to find new CP/T violation effects at Tau-Charm factory (TCF).

TCF is a very good place to test the SM and search new physics phenomena because of its high luminosity and precision [9]. Especially the  $\tau$  sector is a good place to seek for non-SM CP/T violation effects because in the SM CP violation in lepton sector occurs only at multi loop level and is way below any measurable level in high energy experiments, only non-SM sources of CP/T non-conservation may contribute and another reason is that  $\tau$  has abundant decay channels with sizable branching ratio, which can be used to measure CP/T violation. Furthermore, the production-decay sequences of  $\tau$  pair by electron-positron annihilation is also favored. The reason is as the following: (i)  $\tau$  pair production by electron-positron annihilation is a purely electroweak process and can be perturbatively calculated; (ii) For the unpolarized electron-positron collision, its initial state is CP invariant in the c.m. frame; (iii) when the electron and/or positron beams are longitudinally polarized, the initial state is still effectively CP even, which presents extra chances to detect possible CP violation. To detect the possible CP/T violation, one can either compare certain decay properties of  $\tau^-$  with corresponding CP/T conjugations, or measure some CP/T-odd correlation of momentum or spin of the final state particles from  $\tau$  pair decay. These CP/T violating observables can and should be constructed model independently, since normally in non-SM these observables are not well predicted due to the complexity and many free parameters. The sensitivity of the experimental measurement on the possible CP/T violation is determined by the sensitivity of the measurement on momentum, spin or other physical quantities of the final state particles, from them the physical CP/T violating observables are constructed. The better one can measure these quantities, the momenta, for example, the smaller the CP violation phase can be reached. In TCF, it is to expect about  $10^7$   $\tau$  pair in one year, and the precision of measurement on kinematic parameters at  $10^{-3}$ . The statistical and systematic error can be around or below this level. Therefore generally a CP/T violation phase as small as order

of  $10^{-3}$  can be reached at TCF [9]. In a non-SM the CP/T violating phase may appear in various stages of the process of production-decay chain,  $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \text{final particles}$ . We sort them in three cases; (i) CP/T violation is generated in the tree level production process,  $e^+e^- \rightarrow \gamma, Z, X \rightarrow \tau^+\tau^-$ , where X is some new Higgs or gauge bosons, CP/T violating phase appears either in the propagator of X or in the coupling to lepton pairs, and the simplest possibility is X being a neutral Higgs in two or multi-Higgs doublets model. In this case the size of CP/T violation is proportional to the interference between the X exchange and  $\gamma, Z$  exchange processes. Unfortunately for X being Higgs doublet this interference term is proportional to the initial and final states fermion masses  $m_e m_\tau$  as a result of chirality conservation. This factor along contributes a suppression factor  $m_e/m_\tau \sim 3 \times 10^{-4}$  to all CP/T violating observables in this kind of processes at TCF besides other possible suppression factor, like the large mass of X, small coupling between X and leptons. We conclude that it is hopeless to search for CP non-conservation from the tree-level production process at TCF. (ii) CP/T violation is also generated at production stage, but through loop level. The most hopeful cases are that there may exist large electric or weak dipole moment (EDM or WDM) for  $\tau$  lepton, i.e. there are sizable CP/T violation phase at the vertex  $\tau^- - \gamma, Z - \tau^+$ . For this situation the new physical particles beyond SM only appear as virtual particles through loops and the size of CP violation is proportional to EDM or WDM and is not suppressed by other factors, so the point is just whether EDM or WDM of  $\tau$  is large enough to be observed. Generally the Lagrangian describing the CP/T violation in  $\tau$  pair production related to EDM and WDM is

$$L_{CP} = -1/2 i \bar{\tau} \sigma^{\mu\nu} \gamma_5 \tau [d_\tau^E(q^2) F_{\mu\nu} + d_\tau^W(q^2) Z_{\mu\nu}] \quad (1)$$

$F_{\mu\nu}$  and  $Z_{\mu\nu}$  are the electromagnetic and weak field tensors. The momentum transfer at TCF is around 4 GeV, and in LEP experiments it is around the mass of Z boson. Therefore at TCF we expect the contribution from WDM is a factor of  $\frac{4m_\tau^2}{M_Z^2} \simeq 2 \times 10^{-3}$  smaller than

the contribution from EDM, if EDM and WDM at the same order of the magnitude. On the other hand the EDM term is less important at LEP energy. That is the reason why the LEP data constrain more strictly on WDM than EDM of  $\tau$  [10, 11]. We will neglect the WDM contribution from now on in this work. (iii) It is possible that the CP/T violation phase is small in the production process but it is relatively large in the  $\tau$  pair decay processes. The processes like  $\tau$  to neutrino plus light leptons or hadrons through some new bosons exchange at tree level can contribute significantly to CP/T violation observables. Obviously in this situation any CP/T violation effect from loop level is negligible, since any loop effect is at least suppressed by a factor  $\frac{1}{16\pi^2} \frac{m_\tau^2}{M^2}$ , where  $M$  is the mass of some new heavy particles appearing in loops. This factor is smaller than  $10^{-4}$  if  $M$  is heavier than about 20 GeV.

Now let us recall that how one detects CP violation in  $K$  meson decays: One measures the partial widths for a decay channel and compares it with that for the corresponding CP-conjugate decay process. Underlying such a philosophy is the interference between a CP violating phase and a CP conserving strong interaction phase, i.e. CP violation effect is only manifested in the process with strong final state interaction. To observe possible non-CKM CP violation effects in tau decays, however, one has to invoke new methodology in the most cases. The basic reason is that both in production vertex of  $\tau$  pair (EDM of  $\tau$ ) and in some tau decay channels (like pure leptonic decay,  $\pi\nu$ ,  $\rho\nu$  decay channels etc. ), there is no strong interaction phase, caused by hadronic final state interaction, to interfere with possible CP violating phase. So far some efforts have been made to investigate the CP/T violation effects in TCF. Mainly those work are trying to find various ways to measure possible CP/T violation. The simple and very useful method is to construct observables which are CP/T-odd operators being made from momenta of final state particles coming from  $\tau$  pair decay or polarization vector of the initial electron (or both electron and positron) beam [12]. These operators can be used very conveniently to test any CP/T violation from

either EDM of  $\tau$  lepton or from the decay of the  $\tau$  pair without much model dependence. Some of the operators are constructed by considering the reactions

$$e^+(p) + e^-(-p) \rightarrow \tau^+ + \tau^- \rightarrow A(q_-) + \bar{B}(q_+) + X \quad (2)$$

in the laboratory system, where  $A(\bar{B})$  can be identified as a charged particle coming from the  $\tau^- (\tau^+)$  decay. Some CP/T-odd operators (so CPT even, we will not consider CPT-odd operator in this work since it is certainly much smaller violation effect) can be expressed as following [12]

$$O_1 = \hat{p} \cdot \frac{\hat{q}_+ \times \hat{q}_-}{|\hat{q}_+ \times \hat{q}_-|},$$

$$T^{ij} = (\hat{q}_+ - \hat{q}_-)^i \cdot \frac{(\hat{q}_+ \times \hat{q}_-)^j}{|\hat{q}_+ \times \hat{q}_-|} + (i \leftrightarrow j), \quad (3)$$

where  $\hat{p}, \hat{q}$  denote the unit momenta. If the initial electron and/or positron beams are polarized, one can construct some more observables making use of the initial polarization vector. For example a T violating operator

$$O_2 = \vec{\sigma} \cdot \frac{\hat{q}_+ \times \hat{q}_-}{|\hat{q}_+ \times \hat{q}_-|} \quad (4)$$

can be constructed from the electron polarization vector  $\vec{\sigma}$  and momenta of final state particles. If there exists any sizable CP/T violation from EDM of  $\tau$  or in  $\tau$  pair decay vertex, in principle the experimental expectation values of these operators are nonzero. For EDM of  $\tau$  lepton,  $d_\tau$ , the theoretical expectation values of these operators are worked out and expressed only as a function of  $d_\tau$  [13]. Since at TCF the precision of measurement for these operators are at  $10^{-3}$  level, one expects to probe  $d_\tau$  as small as  $\frac{10^{-3}}{2m_\tau} \simeq 10^{-17}$  e-cm. An example is the measurement of  $d_\tau$  or  $d_\tau^W$  in LEP experiment. Expectation value of  $T^{ij}$  operator is directly related to  $d_\tau$  [10],

$$\langle T_{AB}^{ij} \rangle = \frac{E_{cm}}{e} d_\tau C_{AB} \text{diag}(-1/6, -1/6, 1/3). \quad (5)$$

By the term diag means a diagonal matrix with diagonal elements given above,  $E_{cm}$  is the energy at c.m. frame. The proportional constants  $C_{AB}$  depend on the  $\tau$  decay modes, but generally this constant is order of one for all the decay models [12]. The decay channels, which can be measured in experiments, may be classified as  $l-l$ ,  $l-h$  and  $h-h$  classes, here  $l$  is the lighter leptons,  $h$  is charged hadron like  $\pi$ ,  $\rho$  and  $a_1$ . Very impressively, if the initial electron (or both electron and positron) is polarized, one may use the polarization asymmetrized distribution. The distribution is defined as the differential cross section difference between two different polarizations. With this method, a  $d_\tau$  as small as  $10^{-19}$  e-cm can be reached at TCF [13], this corresponds to a sensitivity of  $10^{-5}$  of CP/T violation. Up to now the best experimental bound on  $d_\tau$  is from LEP experimental data, which is used to exclude indirectly the  $d_\tau$  as large as  $10^{-17}$  e-cm [11], so two order of magnitudes improvement on  $d_\tau$  measurement can be achieved at TCF.

Besides the CP/T-odd operator method, several other useful strategy were proposed to test these violation in  $\tau$  decay. 1) C. A. Nelson and collaborators [14] investigated systematically the feasibility of using the so-called stage-two spin-correlation functions to detect possible non-CKM CP violation in the tau-pair production-decay sequence and the corresponding CP-conjugate sequence. The two-variable energy-correlation distribution  $I(E_A, E_B, \Psi)$ , where  $\Psi$  is the opening angle between the final  $A$  and  $B$  particles, is essentially a kinematic consequence of the tau-pair spin correlation which depends on the dynamics of  $Z^0$  or  $\gamma^* \rightarrow \tau^- \tau^+$  amplitude, and of the  $\tau^- \rightarrow A^- X_A$  and  $\tau^+ \rightarrow B^+ X_B$  amplitudes. By including  $\theta_e$  and  $\phi_e$  which specify the initial electron beam direction relative to the final-state  $A$  and  $B$  momentum directions in the c.m. frame of  $e^- e^+$  system, one obtains the so-called beam-referenced stage-two spin-correlation function  $I(\theta_e, \phi_e, E_A, E_B, \Psi)$ . For the  $\gamma^* \rightarrow \tau^- \tau^+$  vertex, there are four complex helicity amplitudes. Hence, the beam-referenced stage-two spin-correlation function constructs four distinct tests for possible CP violation

in  $e^-e^+ \rightarrow \tau^-\tau^+$ . To illustrate the discovery limit in using the beam-referenced stage-two spin-correlation function, Goozovat and Nelson [15] calculated the ideal statistical errors corresponding to the four tests. An advantage of detecting CP violation by use of the stage-two spin-correlation function is that the model independence and amplitude significance of the results is manifest. It is complementary to the greater dynamical information that can be obtained through other approaches, such as from higher-order diagrammatic calculations in the multi-Higgs extensions of the SM. 2) Another strategy to test CP violation in the two-pion channels of tau decay is due to Y.S. Tsai [16], the basic ingredient of which is to invoke a highly polarized tau-pair. Consider the tau-pair production by electron-positron annihilation near threshold. If the initial electron and positron beams are polarized longitudinally (along the same direction), the tau-pair will be produced mainly in the  $S$ -wave, resulting in polarizations of  $\tau^\pm$  both pointing in the same direction as that of the initial beams. Such a polarization is independent of the production angle and the corresponding polarization vector supplies us with an important block to form products with the final particle momenta. By comparing such polarization-vector-momentum products for a specific tau decay channel with those for the corresponding CP-conjugate process, one can perform a series of tests for possible CP violation effects in the tau decay. However, it is impossible to detect a CP violation in the  $\tau \rightarrow \pi\nu_\tau$  decay without violating CPT symmetry. As for the two-pion channel, the existence of a complex phase due to the hadronic final-state interactions, given by the Breit-Wigner formula for the  $P$ -wave resonance  $\rho$ , enables detecting possible non-CKM violation by measuring asymmetry of  $(\mathbf{w} \times \mathbf{q}_1) \cdot \mathbf{q}_2$  without violating the CPT symmetry, where  $(\mathbf{w}$  is the tau polarization vector and  $\mathbf{q}_i$  ( $i=1,2$ ) are the final pion momenta). By limiting the weak interaction to be transmitted only by exchange of spin-one and spin-0 particles, one can know that only the  $S$ -wave part of the amplitude for the exchange of the extra spin-1 particle make contributions to CP violating observables.



A very generic conclusion is that unless two diagrams have different strong interactions phases, one cannot observe the existence of weak phase using terms involving  $\mathbf{w} \cdot \mathbf{q}_1$ . Tsai [17] also points out that T violation can not be detected in the pure leptonic decay without detecting the polarization of the decay lepton. Because it is impossible to construct T-odd operator by the momenta of the initial and final state particles in pure leptonic three body decays. This also implies that with CPT symmetry, one can not detect CP violation in  $\tau$  decay processes with unpolarized  $\tau$ . On the other hand, however, with polarized initial electron and positron beams, one can construct T-odd operators using the momenta and polarization vector of  $\tau$  and the decay lepton. Therefore polarization of initial electron and positron is very desirable for detecting of CP/T violation at TCF. 3) As for the  $\tau \rightarrow (3\pi)\nu_\tau$  decay, it can proceed either via  $J^P = 1^+$  resonance  $a_1$  and the  $J^P = 0^-$  resonance  $\pi'$ . Choi, Hagiwara and Tanabashi [18] investigated the possibility that the large width-mass ratios of these resonances enhance CP-violation effects in the multi-Higgs extensions of the SM. To detect possible CP-violation effects, these authors compare the differential decay width for the  $\tau^- \rightarrow \pi^+\pi^-\pi^-\nu_\tau$  with that for the corresponding CP-conjugate decay process. To optimize the experimental limit, they suggested considering several CP-violating forward-backward asymmetry of differential decay widths, with appropriate real weight functions. 4) To probe possible CP-violating effects in the tau decay with  $K^-\pi^-\pi^+$  or  $K^-\pi^-K^+$  final states, Kilian, Körner, Schilcher and Wu [19] partitioned the final-state phase space into several sectors and constructed some asymmetries of the differential decay widths. As a result, they showed that T-odd triple momentum correlations are connected to certain asymmetries, so their non-vanishing would indicate a possible non-CKM CP violation in the exclusive semileptonic  $\tau \rightarrow$ three pseudoscalar-meson decays.

With these knowledge and results obtained in the previous papers in mind, now the crucial question, which is also the motivation of this work, is whether for CP/T violation

appearing in EDM close to  $d_\tau \sim 10^{-19}$  e-cm and CP/T violation effects in  $\tau$  decay as  $10^{-3}$  are possible values theoretically. If for all possible extensions of the SM, which people can visualize now, with natural parameter choice, these values are much smaller than the theoretically predicted ones, then the effort to search for such small CP/T violation signal at TCF would be not much meaningful, at least from the theoretical point of view. In this paper we are trying to answer this question by investigating various possible mechanisms for generating large EDM of  $\tau$ , CP/T violation in  $\tau$  decay. This paper is organized as the following. In section 2 we review the generation of EDM of  $\tau$  lepton in various popular beyond standard models and stress on what models can produce possible large EDM of  $\tau$ . Following the discussion of EDM, in the section 3 we concentrate on CP/T violation effects from  $\tau$  decay in the beyond standard models. The last section is reserved for some further discussion, and the conclusion on the possibility of finding CP/T violation at TCF is given.

## 2 EDM of $\tau$ lepton

EDM of the lepton  $d_l$  is a dimension-5 operator. It can only be generated from the loop level. Because this operator changes the chirality of the lepton, it must be proportional to a fermion mass. In the SM EDM of lepton is generated from three loop diagrams and is proportional to lepton mass itself, so  $d_l$  is very small [20]. However generally  $d_l$  can be produced from one loop diagrams in beyond standard model. At one loop level, the  $d_l$  can be expressed as

$$d_l \sim \frac{e\lambda}{16\pi^2} \frac{M_F}{V^2} \sin \phi \sim 10^{-18} \left(\frac{\lambda}{1}\right) \left(\frac{M_F}{100\text{GeV}}\right) \left(\frac{100\text{GeV}}{V}\right)^2 \sin \phi \text{ e-cm} \quad (6)$$

where  $M_F$  is some fermion mass,  $V$  is a large scale from intermediate states in the loops and  $\lambda$  denotes other couplings.  $\phi$  is a CP/T violation phase. In the following part we assume maximal CP/T violation phase, i.e.  $\sin \phi \simeq 1$ . From this equation one sees that  $d_l$  can

be at most as large as  $10^{-18} - 10^{-19}$  e-cm if  $\lambda$  is between  $1.0 - 0.1$ . Since  $V$  is a scale around or larger than weak scale, in order to obtain large  $d_l$ ,  $M_F$  must be a large fermion mass such as t quark mass or new heavy fermion masses. For example if  $M$  is the  $\tau$  mass then  $d_\tau$  is smaller than  $10^{-20}$  e-cm which is not detectable at TCF. Same is true for the scale  $V$ . If  $V$  is at TeV scale  $d_l$  is smaller than  $10^{-20}$  e-cm. Although, in principle,  $d_\tau$  is possibly as large as  $10^{-19}$  e-cm, one has to avoid too large EDM of electron  $d_e$  at the same time. Current experimental upper limit on  $d_e$  is about  $10^{-26}$  e-cm. This is a very strong constraint especially when one is expecting large  $d_\tau$ . So in any beyond standard model, two requirements must be satisfied in order to obtain measurable  $d_\tau$ . The first one is that the model must provide  $d_\tau$  at one loop level and  $d_\tau$  is not suppressed by a small fermion mass term, the fermion mass term should be a top quark mass, supersymmetric partner of bosons or other exotic fermion masses. The second one is that the predicted  $d_e$  associated with large  $d_\tau$  is below its current experimental bound. These two conditions altogether exclude most of beyond standard models which can provide large enough  $d_\tau$  observable for TCF. We will see from the following discussion that many beyond standard models do not satisfy the two requirements.

Usually EDM of lepton is generated from one loop diagrams in extension of the SM. Fig. 1 is a typical one loop diagram for the lepton EDM. The virtual particles are scalar or vector boson  $S$  and fermion  $F$  in the loop. Photon is attached to the charged intermediate particles. The  $d_l$  from this diagram is approximately proportional to the fermion mass  $M_F$  and it is divided by a scale  $V$ , which is larger or equal to  $M_F$ . Besides, there are two more couplings at the vertex  $l - S - F$ . In a practical model there could be many possible virtual bosons and fermions in the loop, but we only consider the dominant contribution here as an order of magnitude estimation. The diagram in Fig. 1 is evaluated as

$$d_i/Q \simeq \frac{|\lambda_i \lambda_i^*|}{16\pi^2} \frac{M_F}{V^2} \xi \sin \phi \quad (7)$$

where  $i = e, \mu, \tau$  denotes three generation leptons and  $Q$  is the electric charge of the virtual particles.  $\xi$  is an order of one factor from the loop integral. Eq. (7) is true up to a factor of order one. And there should be a logarithmic dependence on  $\frac{M_F}{V}$  in  $\xi$ , which is slowly varying.

In order to obtain measurable  $d_\tau$  and avoid too large  $d_e$ , one needs a large  $M_F$  as discussed before and  $\lambda, \lambda'$  must be around order of one for  $\tau$  but much small (smaller than about  $10^{-3}$ ) for electron. We systematically investigate and review most of the popular extensions of the standard model and point out that the following type of models can fulfill the requirements.

**Scalar leptoquark models**[21] CP violation effect in  $\tau$  sector for the models are recently discussed extensively by some authors [18, 22]. It is particularly interesting for generating a large  $d_l$ . These are the models which do not need to introduce additional fermion. Because the top quark mass is large, it is possible to generate a large  $d_\tau$  through coupling of  $\tau$ , top quark and the corresponding leptoquark.  $d_e$  could be small enough due to the coupling of electron, top quark and leptoquark is independent of that for  $d_\tau$ . So long as there is a relative large hierarchy for the couplings for different generations, the two requirements can be satisfied.

There are five types of scalar leptoquarks which can couple to leptons and quarks. We denote them by  $S_1, S_2, S_3, S_4$  and  $\vec{S}_5$ . Their quantum numbers under standard gauge group transformation are  $(3, 2, \frac{7}{3}), (3, 1, -\frac{2}{3}), (3, 2, \frac{1}{3}), (3, 1, -\frac{7}{3})$  and  $(3, 3, -\frac{2}{3})$  respectively. The Yukawa coupling terms are therefore given by

$$L_1 = (\lambda_1^{ij} \bar{Q}_{Li} i\tau_2 E_{Rj} + \lambda_1'^{ij} \bar{U}_{Ri} l_{Lj}) S_1 + h.c.$$

$$L_2 = (\lambda_2^{ij} \bar{Q}_{Li} i\tau_2 l_{Lj}^c + \lambda_2'^{ij} \bar{U}_{Ri} E_{Rj}^c) S_2 + h.c.$$

$$L_3 = \lambda_3^{ij} \bar{D}_{Ri} l_{Lj} S_3 + h.c.$$

$$L_4 = \lambda_4^{ij} \bar{D}_{Ri} E_{Rj}^C S_4 + h.c.$$

$$L_5 = (\lambda_5^{ij} \bar{Q}_{Li} i\tau_2 \vec{l}_{Lj}^c) \cdot \vec{S}_5 + h.c.$$

Here  $l_L$  and  $Q_L$  are lepton and quark doublets respectively,  $U_R$ ,  $D_R$  and  $E_R$  are singlet quark and lepton respectively. Individually only  $S_1$  and  $S_2$  contribute to the EDM of lepton.

$\xi$  factor in Eq. (7) is evaluated as  $\xi = \frac{2}{3} \ln \frac{M_E^2}{V^2} + \frac{11}{6}$  [22]. Currently the constraints on mass and coupling of leptoquark are relatively weak [23]. For leptoquark coupled only to third generation, its lower mass bound is about 45 GeV with order of unit coupling [23]. This bound is from a leptoquark pair production from LEP experiments. On the other hand with the leptoquark mass at weak scale, the coupling is very weakly bounded too. In fact the coupling could be as large as order of one. If we take  $\lambda^{33}$ ,  $\lambda'^{33}$  as 0.5 and the mass of leptoquark as 200 GeV and assume maximal CP/T violation phase, we estimate that  $d_\tau \simeq 2 \times 10^{-19}$  e-cm, while  $d_e$  is determined by other coupling components, so a small  $d_e$  is not necessary in conflict with a large  $d_\tau$  in this model.

**Models with the fourth generation or other exotic lepton** The SM with fourth generation is another possible model to generate a large  $d_\tau$ . The heavy fourth generation leptons may play a role of the heavy fermion F in the loop. However it is well known that if the fourth generation exists, it must satisfy the constraints from LEP experiments [24]. Here we propose a realistic model for this purpose.

Besides the fourth generation fermions, we also introduce a right-handed neutrino  $\nu_R$  and a singlet scalar  $\eta^-$  with one unit electric charge [25]. The new interaction terms are

$$L = \lambda_{ij} l_i^T i\tau_2 l_j \eta^- + \lambda'_i E_{Ri}^T \nu_R \eta^- + M_R \nu_R^T \nu_R + M_i^D \bar{\nu}_{Li} \nu_R + h.c. \quad (8)$$

where  $\lambda_{ij}$  is antisymmetric due to the Fermi statistics.  $M^D$  is the Dirac neutrino mass from standard Higgs vacuum expectation value. In this model three light neutrinos remain massless and the fourth neutrino is massive [26]. The constraints from LEP experiments

and other low energy data can be satisfied so long as  $M_R$  is at weak scale or up and  $M_i^D$  is not much smaller than  $M_R$ . In the one loop diagram contribution to  $d_\tau$ ,  $\eta^-$  appears as the scalar S. The fermion line is two massive neutrinos  $\nu_4$  and  $\nu_H$  in the mass basis and they are related to each other,

$$\begin{aligned}\nu_{L4} &= \cos \theta \nu_4 - \sin \theta \nu_H \\ \nu_R &= \sin \theta \nu_4 + \cos \theta \nu_H\end{aligned}\tag{9}$$

We assume  $\nu_4$  is the lighter neutrino and the dominant contribution is from either  $\nu_H$  or  $\nu_4$  depending on whether  $\nu_H$  is heavier than the mass of  $\eta$ ,  $M_\eta$ .  $d_\tau$  is evaluated as in (7) with  $M_F = M_H \cos \theta \sin \theta$  and  $V \simeq M_\eta$  if  $M_\eta \geq M_H$ ; with  $M_F = M_{\nu_4} \cos \theta \sin \theta$  and  $V \simeq M_H$  if  $M_\eta \leq M_H$ . Choosing  $\lambda_{34} = \lambda'_3 = 1.0$  and  $M_F = 50$  GeV,  $V = 200$  GeV, we have the numerical result  $d_\tau \simeq 10^{-19}$  e-cm. Also in this model a hierarchy on the coupling  $\lambda$  and  $\lambda'$  for different generation is needed to keep small enough  $d_e$ , i.e.  $\lambda_{34} \gg \lambda_{14}$  and  $\lambda'_3 \gg \lambda'_1$ .

Existence of exotic leptons provide another possibility to generate a measurable  $d_\tau$ . It can be realized in horizontal models [27]. With only three standard leptons, it is impossible to obtain large enough  $d_\tau$ , because the largest fermion mass in the loop is  $m_\tau$ . However, with some new heavy leptons this model can provide a large  $d_\tau$ . The constraints from low energy data can be avoided if one assumes that the horizontal interaction is strong between  $\tau$  and the exotic lepton, but it is much weaker in other sectors. Similar result on  $d_\tau$  as for the case with the fourth generation can be obtained.

Finally, we should point out that for our purpose it is clear that some new exotic heavy leptons are needed in the new physics models, however even though there exists some kind of models with some new heavy leptons, they are able to generate  $d_l$  only from two loop diagrams [28], so they may result in interesting  $d_e$  but not  $d_\tau$ .

**Generic MSSM** Generic MSSM contains 63 parameters not including the parameters

in the non-SUSY SM. Fermionic superpartners of the ordinary bosons can be the heavy fermions in the loop diagrams for  $d_l$ . It provides some new sources for CP/T violation. It is well known that the electron and neutron can acquire large EDM [29] in this model. In fact, in order to obey the experimental bounds on  $d_n$  and  $d_e$ , some parameters in the model are strongly restricted [30]. For  $d_l$  generation, it is dominated by photino mediated one loop diagram. Both left- and right-handed sleptons also appear in the loop. The contribution to  $d_l$  from this diagram is proportional to left- and right-handed slepton mixing matrices  $M_{LR} = (A_l - \mu \tan \beta)M_l$ .  $A_l$  is the matrix of soft-SUSY-breaking parameters that appears in the SUSY Yukawa terms of slepton coupling to Higgs doublet. Here  $M_l$  is diagonal mass matrix of lepton mass. Usually it is assumed that  $A_l$  is diagonal and the diagonal elements are not much different for different generation, for example in supergravity inspired model  $A_l$  is universal for three generation [8], therefore one can get  $d_\tau/d_e \simeq m_\tau/m_e$ . Using the experimental limit  $d_e \leq 10^{-26}$  e-cm, one concludes that  $d_\tau \leq 4 \times 10^{-23}$  e-cm [31]. However in the generic MSSM all the elements of  $A_l$  are free parameters, so the above constraint is not necessarily true. For example if for some unknown reason the 33 component of  $A_l$  is much larger than other elements, and  $\mu$  term is much smaller than SUSY breaking scale, then  $d_\tau$  still can be larger than  $10^{-22}$  e-cm and  $d_e$  is in the allowed region. In this case  $d_\tau$  can also be expressed as Eq. (7), but with  $M_F = \tilde{m}_\gamma$ ,  $V = \tilde{m}_\tau^2/M_{LR}$ ,  $\lambda_{33} = \lambda'_{33} = e$  and  $\phi = \arg(M_{LR}^2 \tilde{m}_\gamma)$ . The loop integral  $\xi$  was four times the function calculated some years ago in dealing with  $d_e$  in MSSM known as Polchinski-Wise function [32]. Here  $\tilde{m}_\gamma$  and  $\tilde{m}_\tau$  are photino and the third slepton masses respectively. We estimate that  $d_\tau \simeq 10^{-19}$  e-cm with  $\tilde{m}_\gamma = 100$  GeV and  $V = 200$  GeV.

As for other popular extensions of the SM, we would like to point out here, though they have some new sources of CP/T violation, they can not offer a observable  $d_\tau$  at TCF. These include multi-Higgs doublet model ( including two Higgs doublet model) [3, 33], Left-Right

symmetric model [34], mirror fermion model [35] and universal soft breaking SUSY model [8] etc. In multi-Higgs doublet model electron [36] and neutron [5] may obtain a large EDM close to current experimental bounds through two loop diagrams, but  $d_\tau$  generated in the model is quite below the TCF observable value. The reason is that  $d_\tau$  is proportional  $m_\tau$ , but not a large fermion mass. We estimate  $d_\tau \leq 4 \times 10^{-21}$ e-cm [37] that in this model. For Left-Right symmetric model, Nieves, Chang and Pal [38] find that the upper bound for  $d_\tau$  is  $2.4 \times 10^{-22}$ e-cm. It is the right- or left-handed gauge boson in the loop as the role of  $S$  particle, while right-handed neutrino is the virtual fermion particle in the loop.  $d_\tau$  in this model is proportional to left- and right-handed gauge boson mixing angle. Though it is not suppressed by the small fermion mass (  $M_F$  is a large right-handed neutrino mass), the mixing angle is constrained to be smaller than 0.004 [39] from purely non-leptonic strange decays. It leads to about three order of magnitude suppression. In the mirror fermion model, standard gauge bosons couple to ordinary lepton and the mirror lepton with a mixing angle. It is  $Z$  and  $W$  bosons in the one loop diagrams, the heavy fermion line is the mirror lepton. However the mixing angle in this model is constrained by various experiments [40], and most stringently by LEP data on  $Z \rightarrow \tau^+ \tau^-$  [41]. The constraint from LEP data on the mixing angle is less than about 0.3. The resulting bound is  $d_\tau \leq 2.1 \times 10^{-20}$ e-cm, which is a few times smaller than TCF measurable value. As we have mentioned above in the universal soft breaking SUSY model,  $d_\tau \leq 4 \times 10^{-23}$ e-cm due to the constraint on  $d_e$ . The only alternative situation is discussed above on Generic MSSM in this section.

### 3 CP/T violated $\tau$ decays

As we have pointed out in the introduction, CP/T violation effects in  $\tau$  decays, if observed, must occur at tree level diagrams. That is the interference between the SM  $\tau$  decay processes and new tree level processes of  $\tau$  decays, in which CP/T violation phases appear at the



interaction vertexes, provides the information of CP/T violation in the  $\tau$  sector. Feynman diagrams of these processes can be shown as in the Fig. 2, where  $f_i$ ,  $f_j$  and  $f_k$  are light fermions.  $X$  is a new particle ( scalar or vector boson) which mediates CP/T violating interaction. The size of CP/T violation is always proportional to the interference of the tree level diagrams. We denote the amplitudes for these diagrams as  $A_1$  for W boson exchange diagram,  $A_2$  for other X boson exchange diagrams. The size of CP/T violation in the  $\tau$  decay can be characterized by a dimensionless quantity

$$\epsilon = \frac{\text{Im}(A_1^* A_2)}{|A_1|^2 + |A_2|^2} \quad (10)$$

Practically physical quantity expectation values which are used to reflect CP/T violation, like the expectation values of CP/T-odd operators, difference of a partial decay widths of a  $\tau^-$  decay channel and its conjugate  $\tau^+$  decay channel, are model dependent and generally quite complicate. It needs the detailed information of the new physics model and a lot of parameters enter into the expression. This makes it a very much involved work to write down these quantities in a specific model beyond the SM. And the exact CP/T violation quantity expression written down from a model should be different from the  $\epsilon$  defined above. However as a simple and reasonable estimation, the quantity  $\epsilon$  in Eq. (11) can be used as an indication of how large of CP/T violation may happen at various  $\tau$  decays. Moreover, the amplitude  $A_2$  is usually much smaller than  $A_1$  because so far all the experimental data agree with the SM prediction very well. So  $A_2$  term in the denominator can be neglected. Using  $A_1$  as the amplitude from W boson exchange and  $A_2$  from the new boson X exchange, we estimate its size,

$$\epsilon \sim (4\sqrt{2}G_F)^{-1} \frac{\text{Im}(\lambda\lambda'^*)}{M_X^2} \quad (11)$$

Here  $G_F$  is Fermi constant and  $\lambda$ ,  $\lambda'$  are couplings in  $A_2$ . From Eq. (12) one sees that the size of CP/T violation is determined by the parameter  $\frac{\text{Im}(\lambda\lambda'^*)}{M_X^2}$ . For different models, this

parameter is constrained by some other physical processes. So the possible size of CP/T violation depends on the parameter region which is restricted in a specific model.

In Fig. 2 the final state fermions can be a pair of leptons and quarks besides  $\nu_\tau$ . It corresponds to pure leptonic and hadronic decays respectively. At the quark level, the diagrams with a pair of quarks in the final states denote an inclusive process, it includes all possible hadronic channels originated from quark pair hadronization. Some of the useful hadronic final states like  $2\pi$ ,  $3\pi$ ,  $K\pi$ ,  $K\pi\pi$ ,  $KK\pi$  and  $\rho$ ,  $a_1$  can be used to measure the properties of  $\tau$ . However, it is often difficult to make a reliable quantitative prediction for CP/T violation in exclusive hadronic decay modes, because of the uncertainty in the hadronic matrix elements. On the other hand, for the inclusive cases, one may make a more reliable quantitative estimation due to the fact that one has no need to deal with the hadronization of quarks in this case. In addition, QCD correction should not change the order of the tree level diagram evaluation as the energy scale for  $\tau$  decay processes is around 1GeV. In this section we only deal with the diagrams containing quark pair inclusively, So the CP/T violation size we estimate below is for all the possible hadronic decay channels. In the last section we will comment on our results in exclusive processes. Because of the scale of  $\tau$  mass, its decay products can only be neutrino, electron, muon and hadrons containing only light u,d, s quarks as other heavy quarks are kinematically forbidden. Therefore there are not many possibilities for X particle being the candidate for mediating CP/T violation in the Fig. 2. In fact all the possible choices are the following: X being leptoquark, charged Higgs singlet, doublet and triplet, and double charged singlet. Now we come to discuss these different cases separately.

**Scalar leptoquark models** At tree level it is obvious that only  $S_1$ ,  $S_2$  and  $\vec{S}_5$  contribute to  $\tau$  decays. There are two type of decay processes at quark level,  $\tau \rightarrow \nu_\tau \bar{u}d$  and  $\tau \rightarrow \nu_\tau \bar{u}s$ . The  $\epsilon$  parameter is determined by  $\lambda^{31}\lambda'^{31*}$  and  $\lambda^{32}\lambda'^{31*}$  for these two

type of decays respectively in model 1 and 2 in Eq. (8). For model 5 there is CP/T violation effect only in the second type process, which is determined by  $\lambda^{32}\lambda'^{31*}$ . A direct constraint on these parameters can be obtained through comparing the theoretical value  $\Gamma^{th}(\tau \rightarrow \pi\nu_\tau) = (2.480 \pm 0.025) \times 10^{-13}$  GeV and the measurement value of  $\Gamma^{exp}(\tau \rightarrow \pi\nu_\tau) = (2.605 \pm 0.093) \times 10^{-13}$  GeV [42]. Assuming that real and imaginary part of the coupling  $\lambda\lambda'^*$  are approximately equal, one has from  $\tau \rightarrow \pi\nu_\tau$  [18]

$$\frac{|Im(\lambda^{31}\lambda'^{31*})|}{M_X^2} \sim \frac{|Re(\lambda^{31}\lambda'^{31*})|}{M_X^2} < 3 \times 10^{-6} GeV \quad (12)$$

at  $2\sigma$  level for model one and two. And from  $\tau \rightarrow K\nu_\tau$  a similar result can be obtained for all the three models. Using the theoretical value  $\Gamma^{th}(\tau \rightarrow K\nu_\tau) = (0.164 \pm 0.036) \times 10^{-13}$  GeV [42, 43] and the measurement value  $\Gamma^{exp}(\tau \rightarrow K\nu_\tau) = (0.149 \pm 0.051) \times 10^{-13}$  GeV for the  $\tau \rightarrow K\nu_\tau$  decay width we obtain

$$\frac{|Im(\lambda^{32}\lambda'^{31*})|}{M_X^2} \sim \frac{|Re(\lambda^{32}\lambda'^{31*})|}{M_X^2} < 7 \times 10^{-6} GeV \quad (13)$$

at  $2\sigma$  level. This constraint is less stringent due to the large uncertainties in  $\Gamma^{exp}(\tau \rightarrow K\nu_\tau)$ . With these constraints, one estimates the upper bound of  $\epsilon$  value for the two type of processes as

$$\epsilon(\tau^- \rightarrow \nu_\tau \bar{u}d) \simeq (4\sqrt{2}G_F)^{-1} \frac{Im(\lambda^{31}\lambda'^{31*})}{M_X^2} \leq 4 \times 10^{-2} \quad (14)$$

and

$$\epsilon(\tau^- \rightarrow \nu_\tau \bar{u}s) \simeq (4\sqrt{2}G_F)^{-1} \sin\theta_C \frac{Im(\lambda^{32}\lambda'^{31*})}{M_X^2} \leq 2 \times 10^{-2} \quad (15)$$

where  $\theta_C$  is Cabibbo angle.  $\epsilon(\tau^- \rightarrow \nu_\tau \bar{u}s)$  is proportional to  $\sin\theta_C$  and is smaller than  $\epsilon(\tau \rightarrow \nu_\tau \bar{u}d)$  because this process is Cabibbo suppressed, even though the coupling is less constrained than that of Cabibbo unsuppressed process. From this estimation we expect CP/T violation in these models could be large enough for TCF or in the other words

TCF data can put stronger direct restriction on the parameters of the model. However, if one assumes that all the couplings  $\lambda$  and  $\lambda'$  are at the same size irrespective of the generation indexes, then much more stringent bounds exist. These bounds are obtained from experimental bounds of  $Br(K_L \rightarrow \mu e)$ ,  $Br(\pi \rightarrow e \nu_e(\gamma))$ ,  $Br(\pi \rightarrow \mu \nu_\mu(\gamma))$  and  $\Gamma(\mu Ti \rightarrow e Ti)/\Gamma(\mu Ti \rightarrow capture)$  [18]. They are generally about five order of magnitude smaller than the direct bounds. Therefore the size of CP/T violation is  $\epsilon \leq 4 \times 10^{-7}$  which is far below the capability of TCF.

**Multi-Higgs doublet models (MHD)** With the natural suppression of flavor changing neutral current, it is necessary to have more than two Higgs doublets, so that there are at least two physical charged Higgs particles. CP/T violation may generally happen through the mixing of these charged Higgs particles. We consider a multi-Higgs doublet model, say,  $n$  Higgs doublets. In this model there are  $2(n-1)$  charged and  $(2n-1)$  neutral physical scalars. Since only the Yukawa interactions of the charged scalars with fermions are relevant for our purpose. Following Grossman [44] we write down the Yukawa interactions in fermion mass eigenstates as

$$L_{MHD} = \sqrt{2\sqrt{2}G_F}\Sigma_{i=2}^n[X_i(\bar{U}_L V M_D D_R) + Y_i(\bar{U}_R M_U V D_L + Z_i(\bar{l}_L M_E E_R)]H_i^+ + h.c. \quad (16)$$

Here  $M_U$ ,  $M_D$  and  $M_E$  denote the diagonal mass matrices of up-type quarks, down type quarks and charged leptons respectively.  $V$  is KM matrix.  $X$ ,  $Y$  and  $Z$  are complex couplings which arise from the mixing of the charged scalars and CP/T violation in  $\tau$  decay processes is due to these couplings. How large is the  $\epsilon$  for various  $\tau$  decay channels depends on the values of these parameters. More precisely, in the pure leptonic decays the size of CP/T violation is determined by  $Im(Z_i Z_j^*)$  with  $i \neq j$  and in hadronic decays it is determined by  $Im(X_i Z_j^*)$  and  $Im(Y_i Z_j^*)$ . The three combinations of parameters are constrained by various experiments [44]. The strongest constraint on  $Z$  is from  $e - \mu$  universality in  $\tau$  decay, which gives  $|Z| \leq 1.93 M_H \text{GeV}^{-1}$  for Higgs mass  $M_H$  around 100GeV.

$Im(XZ^*)$  is bounded from above from the measurement of the branching ratio  $Br(B \rightarrow X\tau\nu_\tau)$ ,  $Im(XZ^*) \leq |XZ| \leq 0.23M_H^2\text{GeV}^{-2}$  if  $M_H \leq 440\text{ GeV}$ . Finally a upper bound is given as  $Im(YZ^*) \leq |YZ| \leq 110$  from the experimental data of the process  $K^+ \rightarrow \pi^+\nu\bar{\nu}$ . This bound is obtained for t quark mass at 140 GeV [44] and  $M_H = 45\text{GeV}$ , however for a different  $M_H$ , say 100GeV, this bound is expected not to change much. With these bounds we can estimate CP/T violation size of  $\tau$  leptonic and hadronic decays. For the leptonic decay  $\tau \rightarrow \mu\nu\bar{\nu}$ , we have the quantity

$$\epsilon \simeq \frac{1}{2} \frac{Im(ZZ^*)m_\mu m_\tau}{M_H^2} \cdot \frac{m_\mu}{m_\tau} = \frac{1}{2} \frac{m_\mu^2}{M_H^2} Im(ZZ^*) \leq 2 \times 10^{-2}. \quad (17)$$

Here the additional factor  $\frac{m_\mu}{m_\tau}$  comes from the interference of left- and right-handed muon lines in the final states. So we expect that CP/T violation effect in the process  $\tau \rightarrow e\nu\bar{\nu}$  is suppressed by a factor  $m_e/m_\mu$  and is negligible. For the hadronic decay  $\tau \rightarrow \bar{u}d\nu$  we have

$$\epsilon \simeq \frac{1}{2} \frac{m_d \bar{m}_d}{M_H^2} Im(XZ^*) \leq 3 \times 10^{-4}, \quad (18)$$

With the current  $d$  quark mass  $m_d = 7\text{ MeV}$  and the dynamical  $d$  quark mass  $\bar{m}_d = 300\text{ MeV}$ . For hadronic decay  $\tau \rightarrow \bar{u}s\nu$  similar result is obtained

$$\epsilon \simeq \frac{1}{2} \frac{m_s \bar{m}_s}{M_H^2} Im(XZ^*) \leq 1.5 \times 10^{-3} \quad (19)$$

Here we use current and dynamical  $s$  quark masses as 150 MeV and 400 MeV respectively. In summary, in multi-Higgs doublet model CP/T violation effect is possibly as large as order of  $10^{-3}$  for exclusive hadronic decays and It could be even close to  $10^{-2}$  in pure leptonic decay to  $\mu$  and neutrinos.

**Other extensions of the SM for pure leptonic decays** Besides leptoquark and Higgs doublet, there are three other kind of scalars which can couple to leptons. We denote  $l$  as a lepton doublet and  $E$  as a singlet lepton. Two  $l$  can combine to a charged singlet or a triplet. Two  $E$  can combine to a double charged singlet. Corresponding to

these three cases one can introduce a charged singlet scalar  $h^-$ , triplet scalar  $\Delta$  and double charged scalar  $K^{--}$ . However  $K^{--}$  only induce a lepton family- number-violating process  $\tau \rightarrow 3l$ . There is no diagram corresponding SM contribution, so there is no CP/T violation mediated by this particle. Also the branching ratio ( $\leq 10^{-5}$ ) for this decay is much smaller than TCF reachable CP/T violation precision  $10^{-3}$ . In principle if there exists more than one  $h$  or  $\Delta$ , CP/T violation can be induced by the interference of the W exchange diagram and  $h$  or  $\Delta$  exchange diagram in the process  $\tau \rightarrow l\bar{\nu}\nu$  with  $l = e, \mu$ . Now let us discuss these two possibilities in details. We can write down the new interaction terms which couple the new scalar particles to leptons as the following

$$L_h = \frac{1}{2}f_{ij}l^T_i C i\tau_2 l_j h + h.c. \quad (20)$$

$$L_\Delta = \frac{1}{2}g_{ij}l^T_i C i\tau_2 \vec{l}_j \vec{\Delta} + h.c., \quad (21)$$

where  $C$  is the Dirac charge conjugation matrix and  $f_{ij}$  is antisymmetric,  $g_{ij}$  is symmetric due to Fermi statistics.  $\epsilon$  parameter for these singlet and triplet models are given by,

$$\epsilon_h \simeq (4\sqrt{2}G_F)^{-1} \frac{Im(f_{\tau l}f_{l\tau}^*)}{M_h^2} \quad (22)$$

in singlet model and

$$\epsilon_\Delta \simeq (4\sqrt{2}G_F)^{-1} \frac{Im(g_{\tau l}g_{l\tau}^*)}{M_\Delta^2} \quad (23)$$

in triplet model respectively.

For the singlet model we assume that  $f_{e\mu}$  is considerably smaller than  $f_{\tau l}$ , so that one does not need to readjust the Fermi constant  $G_F$ . This assumption is also consistent with the constraint set by universality between  $\beta$  and  $\mu$  decay [25, 45]. The parameter  $\frac{Im(f_{\tau l}f_{l\tau}^*)}{M_h^2}$  is constrained only by the measurement of  $\tau$  leptonic decays. At  $2\sigma$  level (which is about  $2 \sim 3\%$  precision) we estimate approximately  $\frac{Im(f_{\tau l}f_{l\tau}^*)}{M_h^2} \leq 10^{-6} \text{ GeV}^{-2}$  [46]. It implies that

$$\epsilon_h \simeq (4\sqrt{2}G_F)^{-1} \frac{Im(f_{\tau l}f_{l\tau}^*)}{M_h^2} \leq 1.4 \times 10^{-2} \quad (24)$$

with  $M_h = 100$  GeV. Therefore in this model there is a possibility that CP/T violation effect may show up with a size reachable at TCF in pure leptonic decay channels.

For the triplet model the direct constraint is also from the measurement of pure leptonic decays. The same result is obtained as that in the singlet model, i.e.  $\frac{Im(g_{\tau l} g_{l\tau}^*)}{M_h^2} \leq 10^{-6}$  GeV<sup>-2</sup>. As the result of this constraint one has

$$\epsilon_h \simeq (4\sqrt{2}G_F)^{-1} \frac{Im(g_{\tau l} g_{l\tau}^*)}{M_\Delta^2} \leq 1.4 \times 10^{-2} \quad (25)$$

with  $M_\Delta = 100$  GeV. However, in this model the new interactions will induce lepton family number violating decay  $\tau \rightarrow 3l$  and  $\mu \rightarrow 3e$  through exchange of the double charged scalar particle  $\Delta^{--}$ . Without seeing any signal, one obtains some approximate bounds on the coupling constants as the following [47]

$$\frac{|g_{\mu e} g_{ee}^*|}{M_\Delta^2} \leq 5 \times 10^{-12} \quad (26)$$

and

$$\frac{|g_{\tau l} g_{ll}^*|}{M_\Delta^2} \leq 10^{-8} \quad (27)$$

for  $M_\Delta = 100$  GeV. If one assumes that all the couplings  $g_{ij}$  are at the same order of magnitude, then these bounds will restrict the CP/T violation size far below the ability of TCF. Again we see some hierarchies on the couplings are needed for this model to give rise observable CP/T violation effects. Additionally in the triplet model one has to avoid the restriction from neutrino mass generation [48]. If neutrino develops a mass at tree level, either the couplings or the vacuum expectation value of the neutral component of the triplet  $\Delta^0$  are extremely small. The natural way to deal with this problem is to impose some symmetry on this model. An example is to introduce a discrete symmetry:

$$l \rightarrow il; \quad E \rightarrow iE \quad \Delta \rightarrow -\Delta \quad (28)$$

With this symmetry,  $\Delta^0$  will never develop a nonzero vacuum expectation value, therefore the couplings are not constrained by the neutrino mass generation.

## 4 Discussion and conclusion

In this work we systematically investigate the possibility of finding CP/T violation in the  $\tau$  sector with TCF. The origin of CP/T violation is from the extensions of the SM. We discuss most of the popular beyond the SM and present the models which may give rise large CP/T violation in  $\tau$  sector through either EDM or decay of  $\tau$  lepton. Before making our conclusion, some interesting points should be further discussed or emphasized. (1) Polarization of initial electron and/or positron is very desired for our purpose. First with polarization the precision of measurement of EDM will be increased by about two order of magnitude, as  $10^{-19}$ e-cm, which is used through this work. Without polarization, from our above discussion one sees that we have no hope to expect a detectable EDM of  $\tau$  at TCF. Secondly in some decay channels without final state interaction, like pure leptonic decays and two body decays  $\pi\nu_\tau$  etc., polarization is needed to search for CP/T violation occurring at  $\tau$  decay vertex. With unpolarized electron and positron beams the CP/T violation could only be detected using channels with final state interaction phase, like  $2\pi\nu_\tau$  etc. (2) For the hadronic decay we only consider inclusive processes. The advantage of inclusive process is that one does not need not to consider the hadronization of quarks, which may bring in large uncertainties in the estimation. And the event number in inclusive process is larger than that in certain exclusive processes. However we should mention that for certain exclusive decays the CP/T violation parameter  $\epsilon$  can be larger than that in inclusive decay. One example is from the multi-Higgs double model. We estimate that  $\epsilon \leq 3 \times 10^{-4}$  for the decay  $\tau \rightarrow \bar{u}d\nu_\tau$ . Here we may also consider the exclusive decay  $\tau \rightarrow 3\pi\nu_\tau$  contributed by  $a_1$  and  $\pi'$  resonances. Compared to inclusive decay, the  $\epsilon$  parameter is larger by a factor of (using



current algebra relation)

$$\frac{\langle o|\bar{u}_L d_R|\pi' \rangle}{\langle o|\bar{u}_L \gamma_0 d_L|\pi' \rangle} \simeq \frac{m_{\pi'}}{m_u + m_d} \simeq 100. \quad (29)$$

So  $\epsilon \leq 3 \times 10^{-2}$  is obtained. However on the other hand the event number decreases by a factor of

$$\frac{f'_\pi}{f_\pi} \frac{Br(\tau \rightarrow \pi \nu_\tau)}{Br(\tau \rightarrow hadron + \nu_\tau)} \simeq 10^{-2} \quad (30)$$

Here  $f_{\pi'} = 5 \times 10^3$  GeV is used. Therefore statistical error increases by about 10 times. In the other words the measurement precision at TCF for this channel is about  $10^{-2}$ . As the result, at  $2\sigma$  level  $\epsilon \simeq 3 \times 10^{-2}$  is observable. This estimation agrees with the exact result of reference [18]. (3) Obviously the numerical result we obtained above is quite crude. More accurate estimation is necessary in the future. For instance through this paper we assume that EDM as large as  $10^{-19}$ e-cm and  $\epsilon$  as large as  $10^{-3}$  can be observed. This of course is a rough estimation. To be more precise, Monte Carlo simulation is needed, which will tell us more confidently how large CP/T violation is able to be observed at TCF. Especially the Monte Carlo simulation on EDM of  $\tau$  will give us a quite clear result, because in this case the  $d_\tau$  is the only parameter we should take care. All the model dependence is included in it. Recently a group of people analyzed the data from BEPC experiments to set bound on the T-violating effect for  $\tau$  system [49]. Following the suggestion by T.D. Lee, they considered the pure leptonic  $\tau^\pm$  decays to  $e^\pm \mu^\mp$  plus neutrinos in the final states. The T-violating amplitude

$$A = \langle \hat{p}_e \cdot (\hat{p}_1 \times \hat{p}_2) \rangle_{average} \quad (31)$$

is measured, where  $\hat{p}_e$  is the unit momentum vector of the initial electron beam,  $\hat{p}_1$  and  $\hat{p}_2$  are the unit momenta of the final state electron and muon respectively. Totally 251 events are analyzed and it results in

$$A = -0.097 \pm 0.039 \pm 0.135 \quad (32)$$

This result agrees with no T-violation as expected from our previous discussion on pure leptonic  $\tau$  decays. (4) In order to generate detectable large CP/T violation effects, we know from our investigation that there must exist new physics and the new physics scale is not far above the weak scale. Therefore if there is an observable CP/T violation effect in  $\tau$  sector at TCF, the associated new physics phenomena should be observed at high energy experiments, like LHC and LEP II experiments. It is interesting to see if the new particles predicted by the various models we have discussed in this paper are indeed detectable in these high energy experiments. (5) Precise measurement of the pure leptonic decay is another way to test the new physics responsible for CP/T violation. Since if there is CP/T violation effect at level of  $10^{-3}$ , the  $\tau$  leptonic decay width must deviate from the SM prediction at the same level. So we expect to observe the deviation by measuring the branching ratio of the pure leptonic decay. However it is not true *vice versa*, since a deviation of leptonic branching ratio from that of the SM does not necessarily indicate CP/T violation.

Finally we come to our conclusion. There exists the possibility that CP/T violation in  $\tau$  sector is large enough to be discovered at TCF, although for this large violation effect some specific new physics phenomena beyond the SM are needed and the parameter spaces of the models are strongly restricted.

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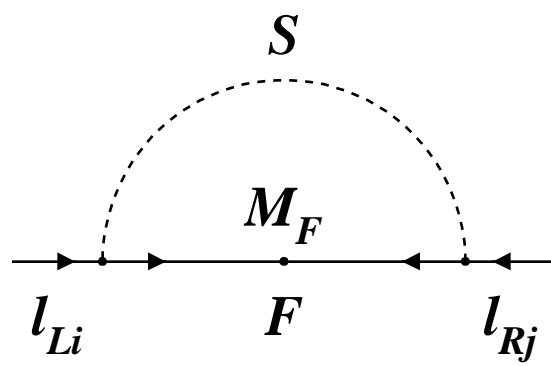
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### Figure Caption

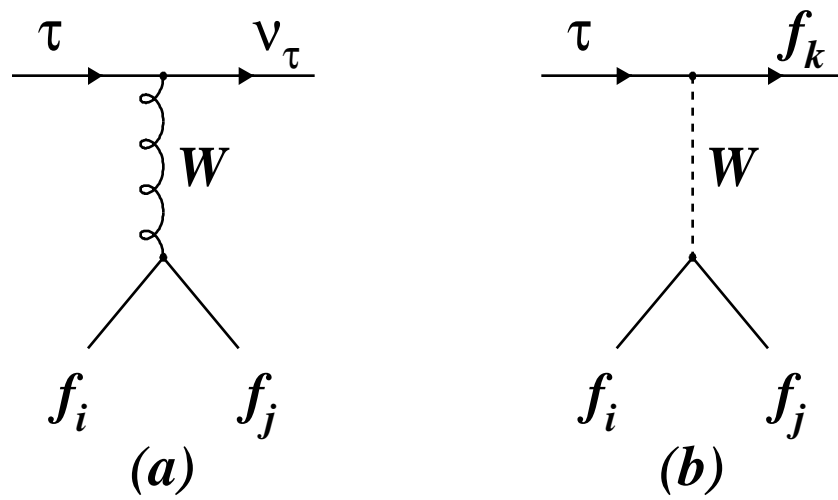
Fig.1 One loop diagram for lepton EDM generation, where  $F$  is heavy fermion and  $S$  is the new boson. Photon line is attached to charged particles in the loop.

Fig.2 The diagrams for  $\tau$  decay. (a) is the contribution from the SM and (b) is the contribution from new boson exchange.



*Fig.1*





***Fig. 2***